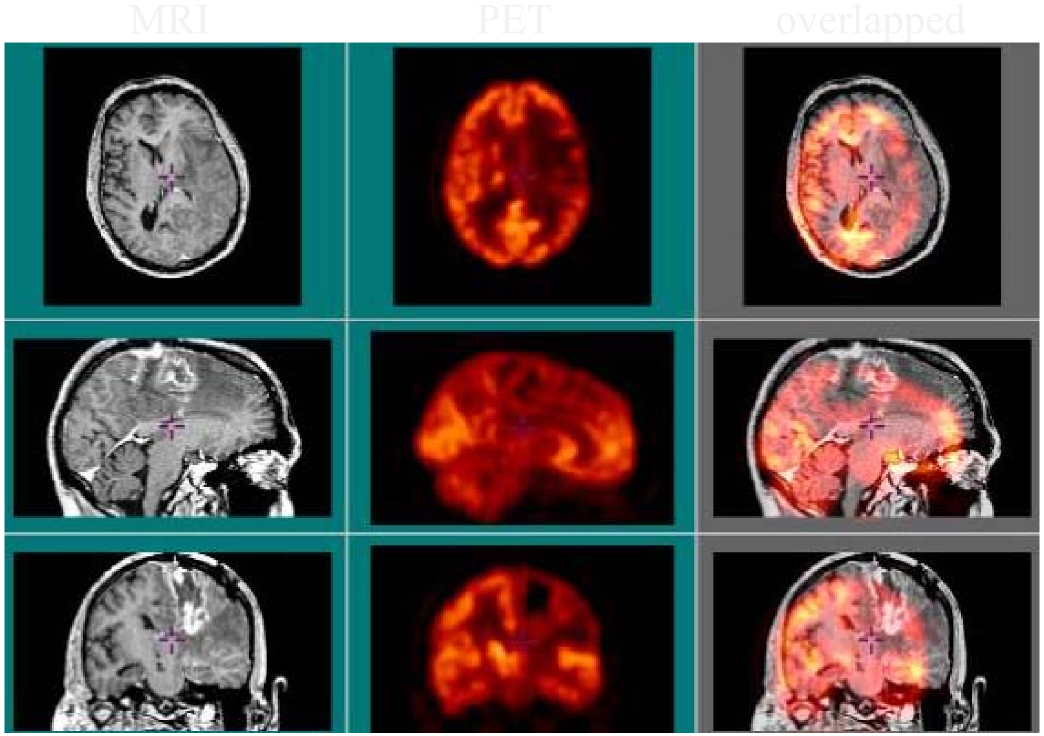
# Image registration

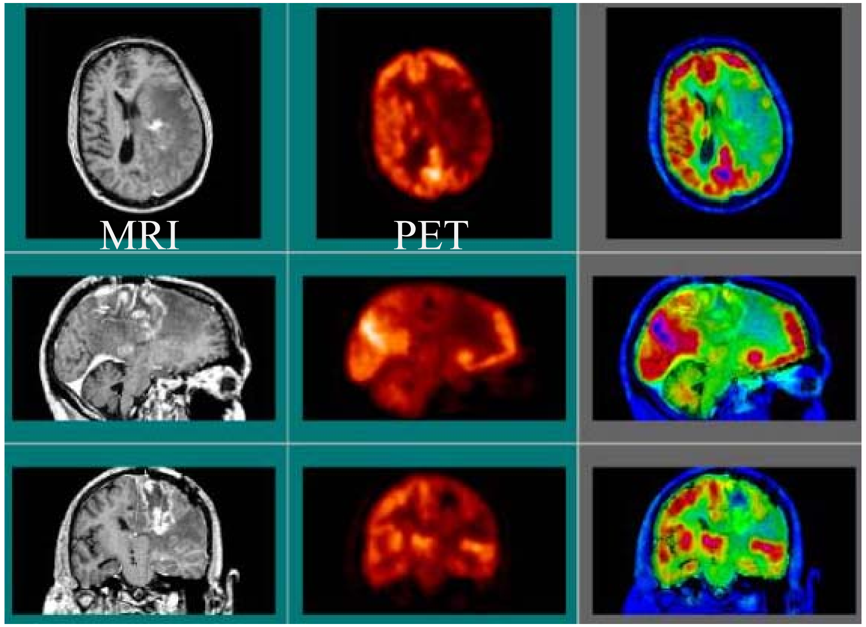
莊子肇 副教授
中山電機系

# Why image registration?

- For the subject(s) scanned at
  - different times
  - different perspectives
  - ….



# Why image registration?




- Registration: transforming different sets of data into an identical coordinate system
  - Google map/earth 用過吧…

# Medical image registration

- Possible applications
  - Motion correction
  - Combine structural and metabolic information (multi-modality analysis)
  - Inter-subject comparison
  - Image-guided surgery
  - …

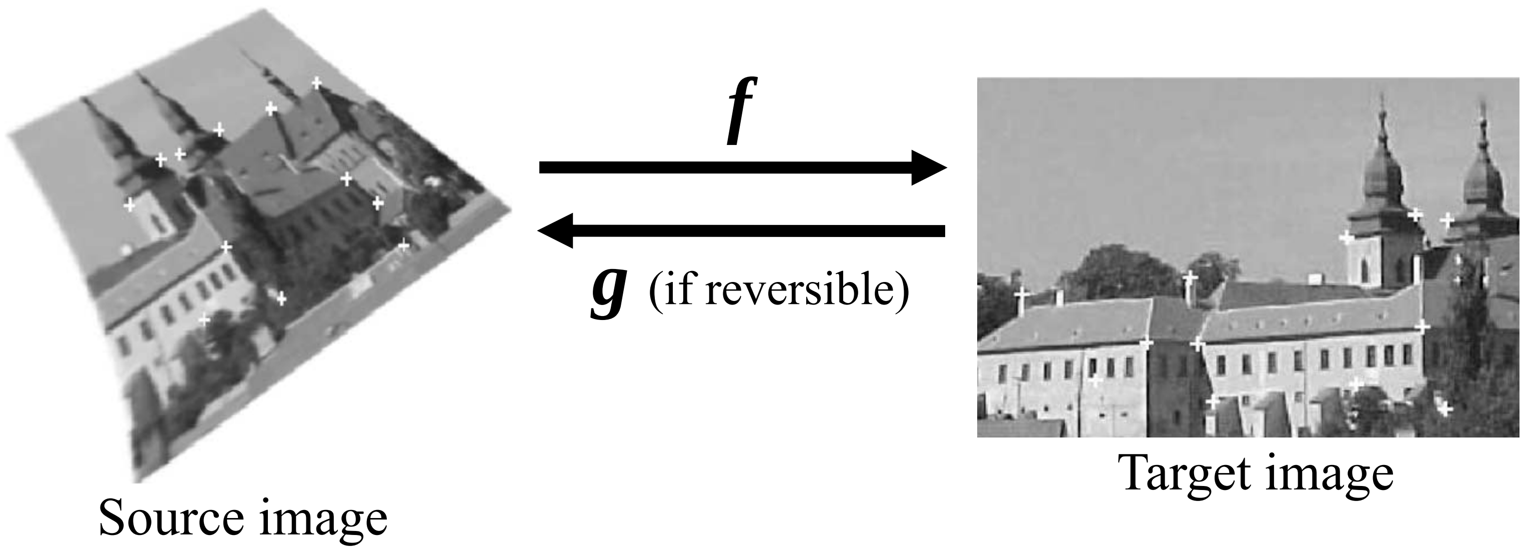
# Medical image registration

- A basic image similarity-based method consists of:
  - Transformation model
  - Image similarity metric
  - Optimization algorithm

# Registration models

# Registration models

- Rigid-body transformation
- Affine transformation
- Elastic (non-rigid) transformation



# Rigid transformation

- Translation and rotation
  - A pixel-by-pixel rigid-body transformation (2D or 3D) mapping from a point vector *x* to *x’* is defined as:

$$x' = Rx + t$$

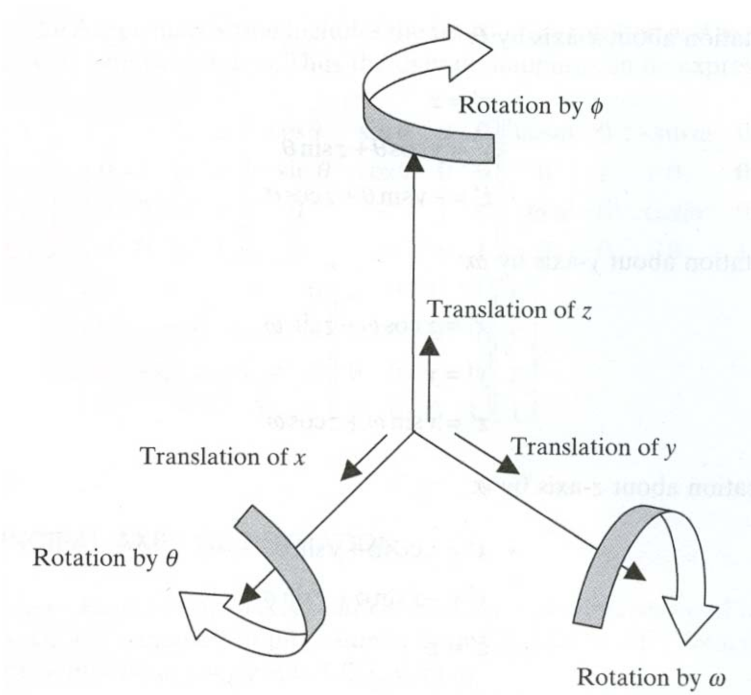
*R*: rotation matrix

*t*: translation vector (Ex: $[p, q, r]^T$)

# Rigid transformation- rotation

- Rotation matrix ($R$)

$$R = R_\phi R_\omega R_\theta = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\omega & 0 & -\sin\omega \\ 0 & 1 & 0 \\ \sin\omega & 0 & \cos\omega \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix}$$



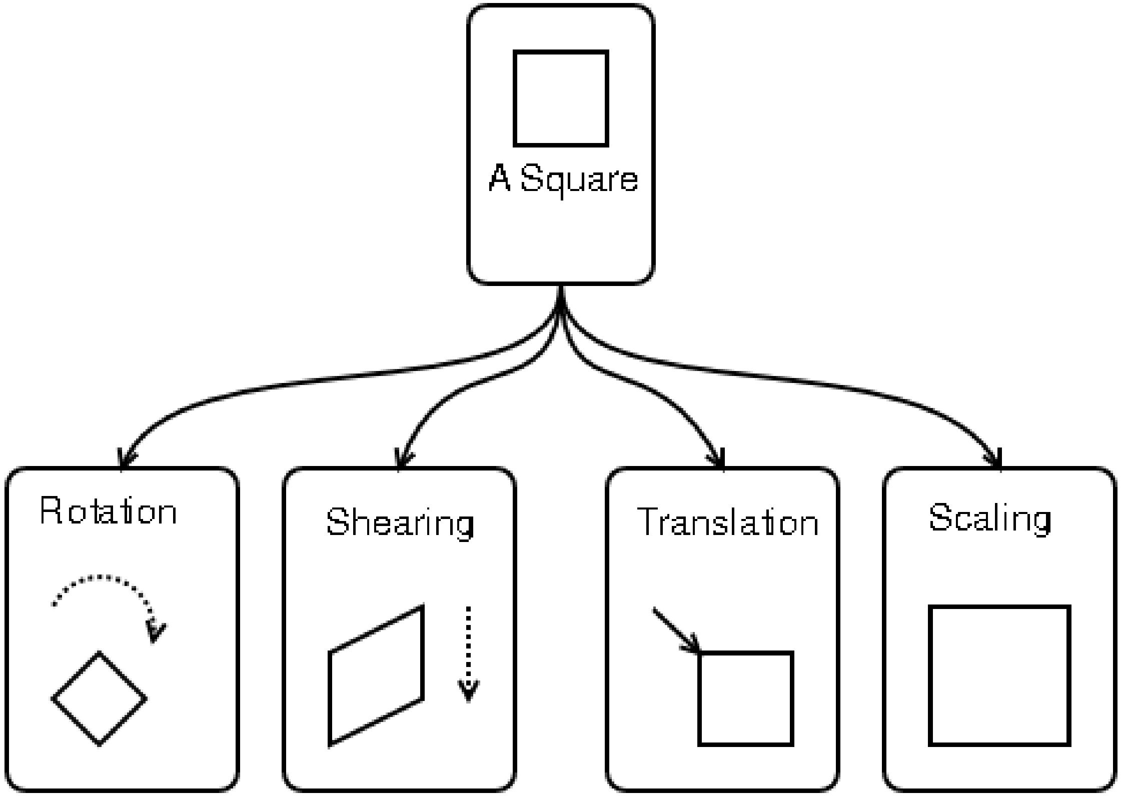
# Affine transformation

- Translation, rotation, scaling, and shear.
- The Affine transformation can be expressed as:

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & p \\ 0 & 1 & 0 & q \\ 0 & 0 & 1 & r \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\phi & \sin\phi & 0 & 0 \\ -\sin\phi & \cos\phi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\omega & 0 & -\sin\omega & 0 \\ 0 & 1 & 0 & 0 \\ \sin\omega & 0 & \cos\omega & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a & s_{xy} & s_{xz} & 0 \\ s_{yx} & b & s_{yz} & 0 \\ s_{zx} & s_{zy} & c & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

# Affine transformation



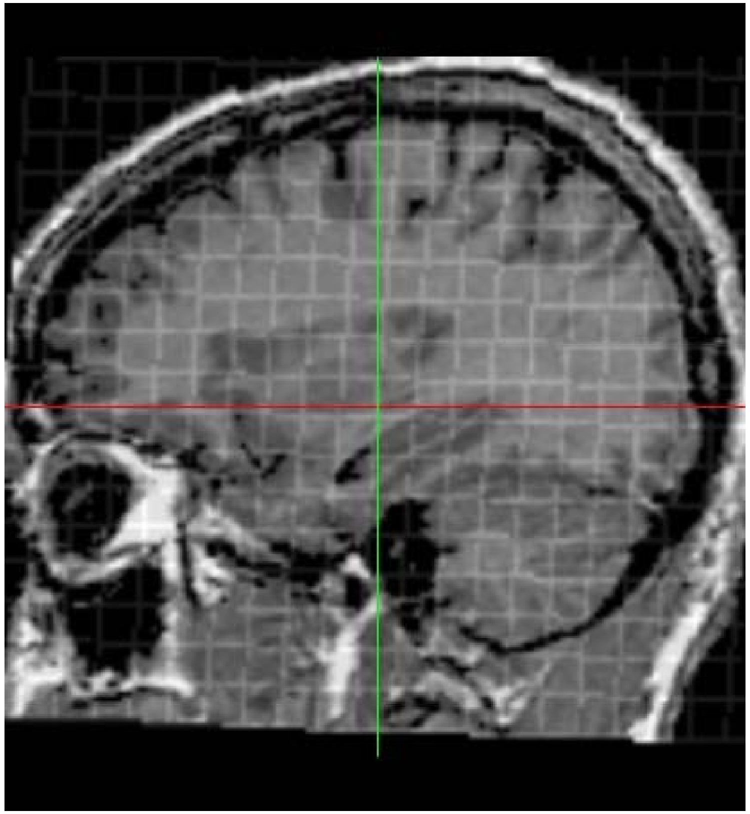
# Affine transformation

- Also called “shape-preserving mapping”
- A linear model
  - Straight lines are mapped onto straight lines after transformation.
  - Can be expressed by a 4×4 matrix
- Global mapping is usually adapted to align the source image to the target (reference).

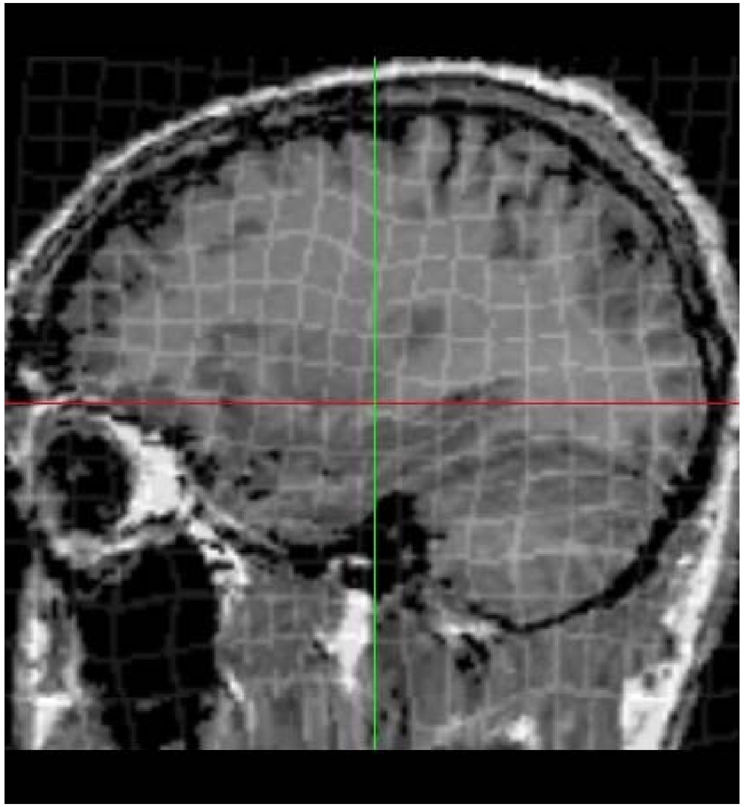
# Elastic transformation

- Needed for inter-subject registration and distortion correction
  - Images are viewed as a rubber sheet.
- Non-linear
  - Can not be represented using constant matrices
- The degree of non-rigid has to be controlled.
  - Too much flexibility in the transformation can lead to undesirable results

# Affine vs Non-Rigid

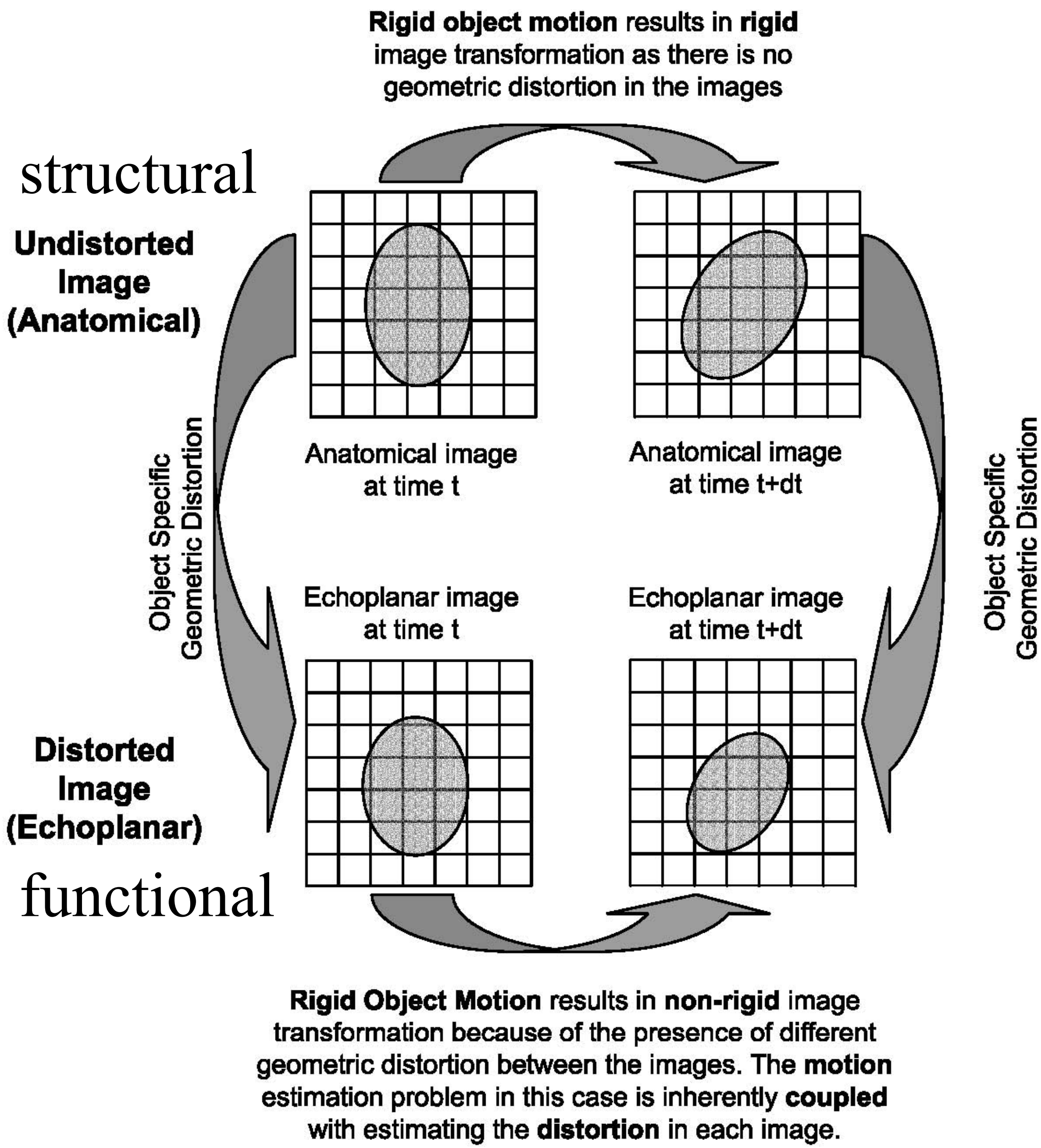


Affine: 12 parameters

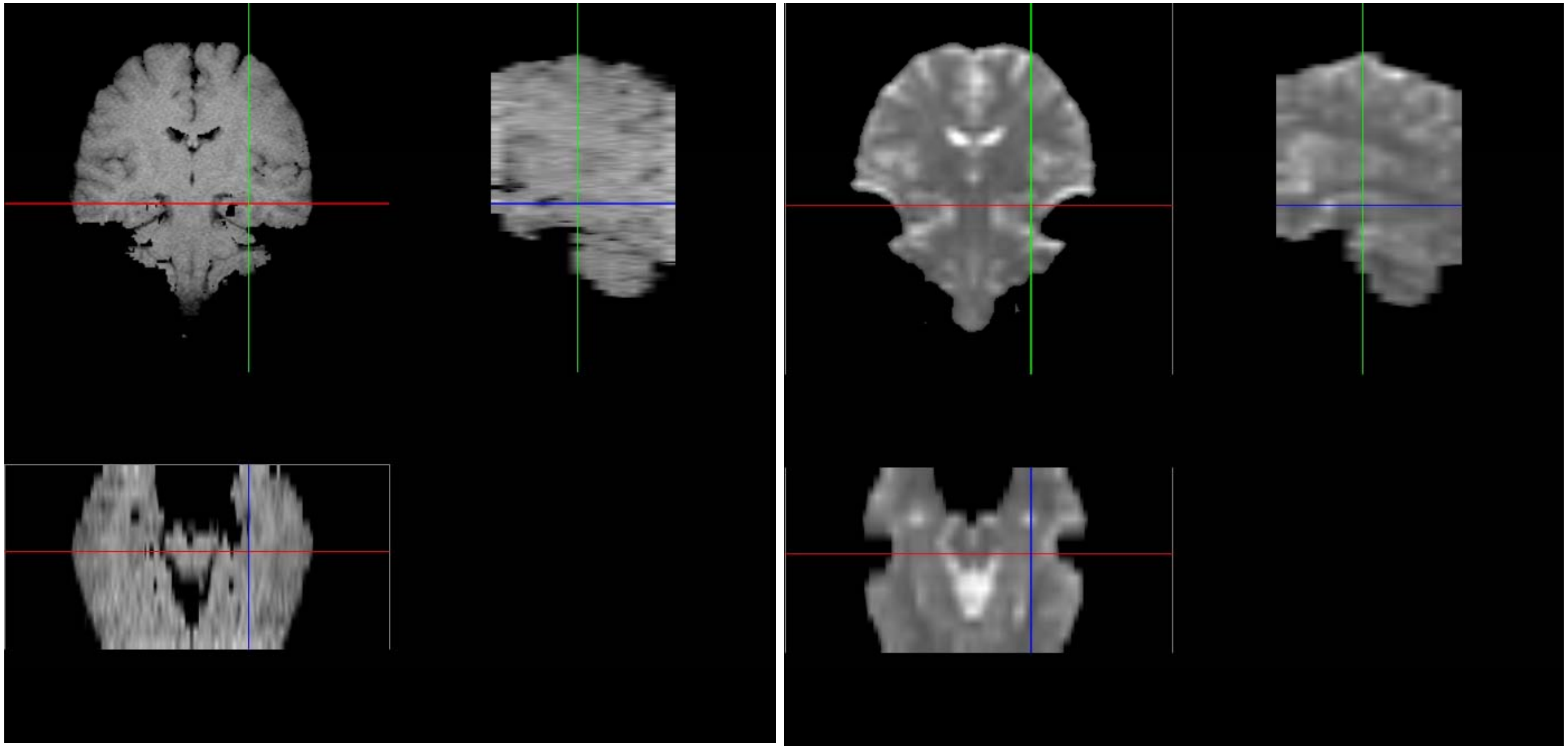


Non-Rigid ~ 2000 parameters

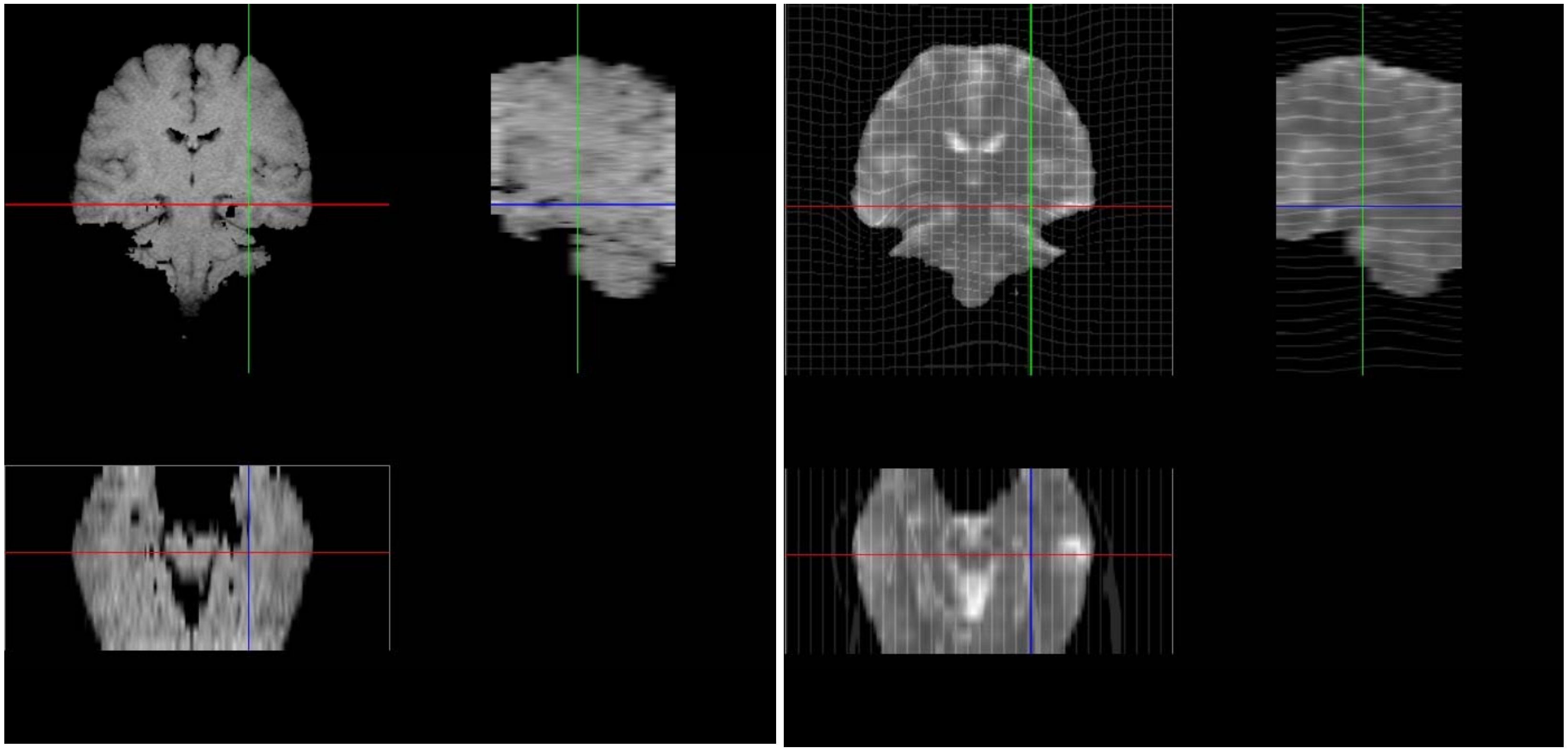
Example of registration: fMRI study



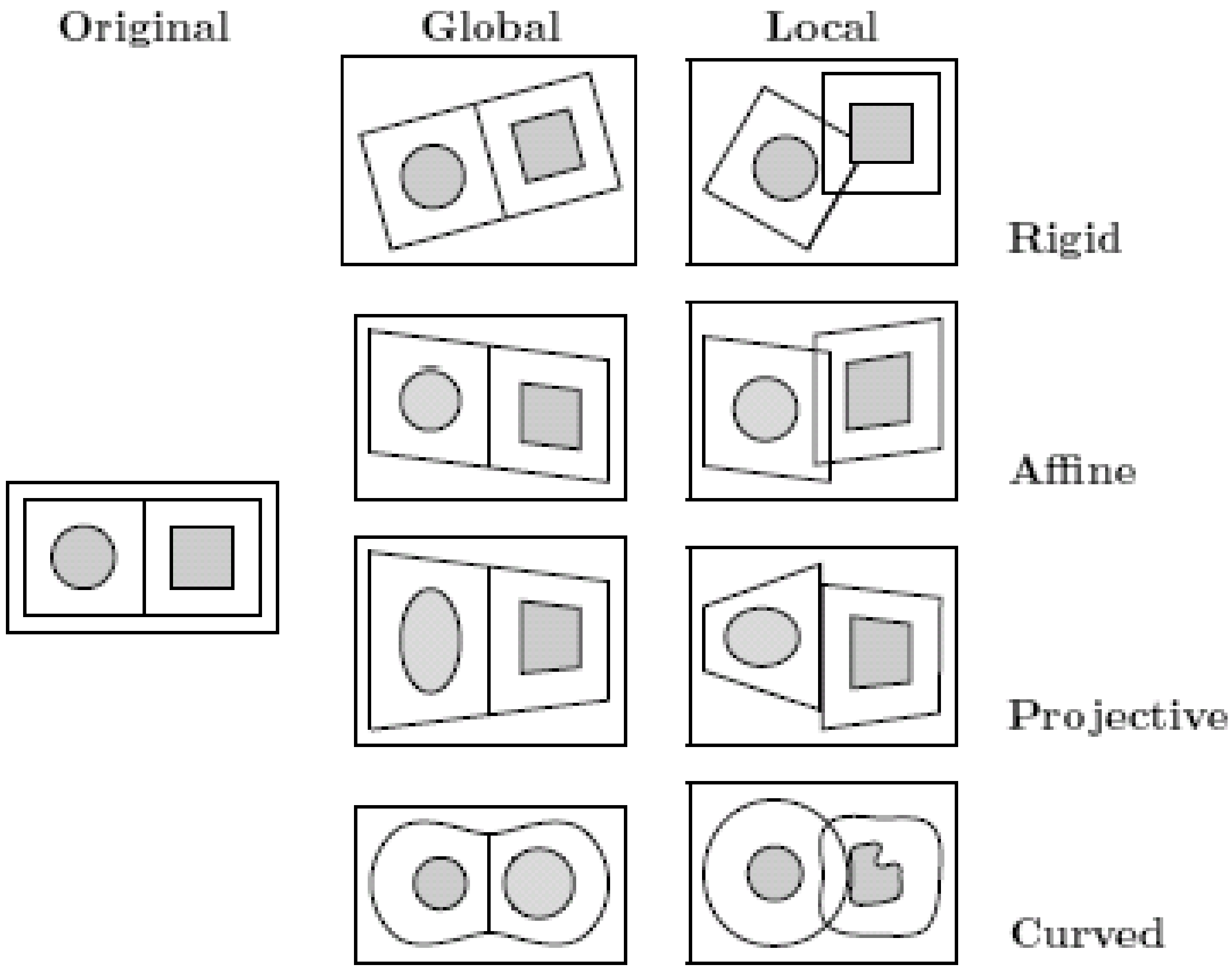
# Example of registration -- Before



# Example of registration -- After



# Global vs. local transformation



# Similarity metric

Intensity-based method

Feature-based method

# Intensity-based methods (1)

- Sum of squared differences
  - Straightforward and easy
  - Only valid for data of the same modality
  - Source and target may NOT have the same field of view (FOV)

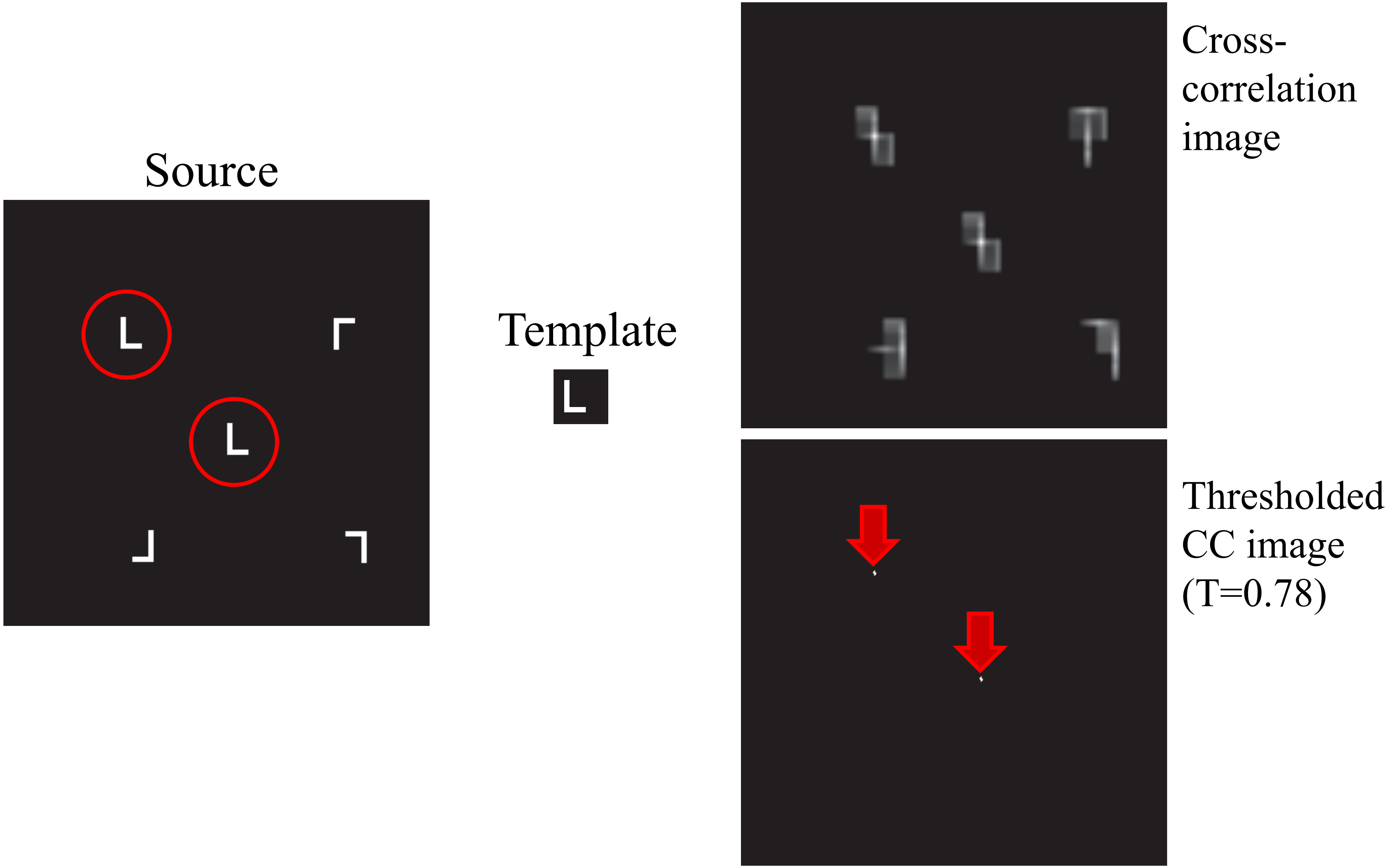
# Intensity-based methods (2)

- Normalized cross correlation
  - Linear relationship between the intensities of the two images

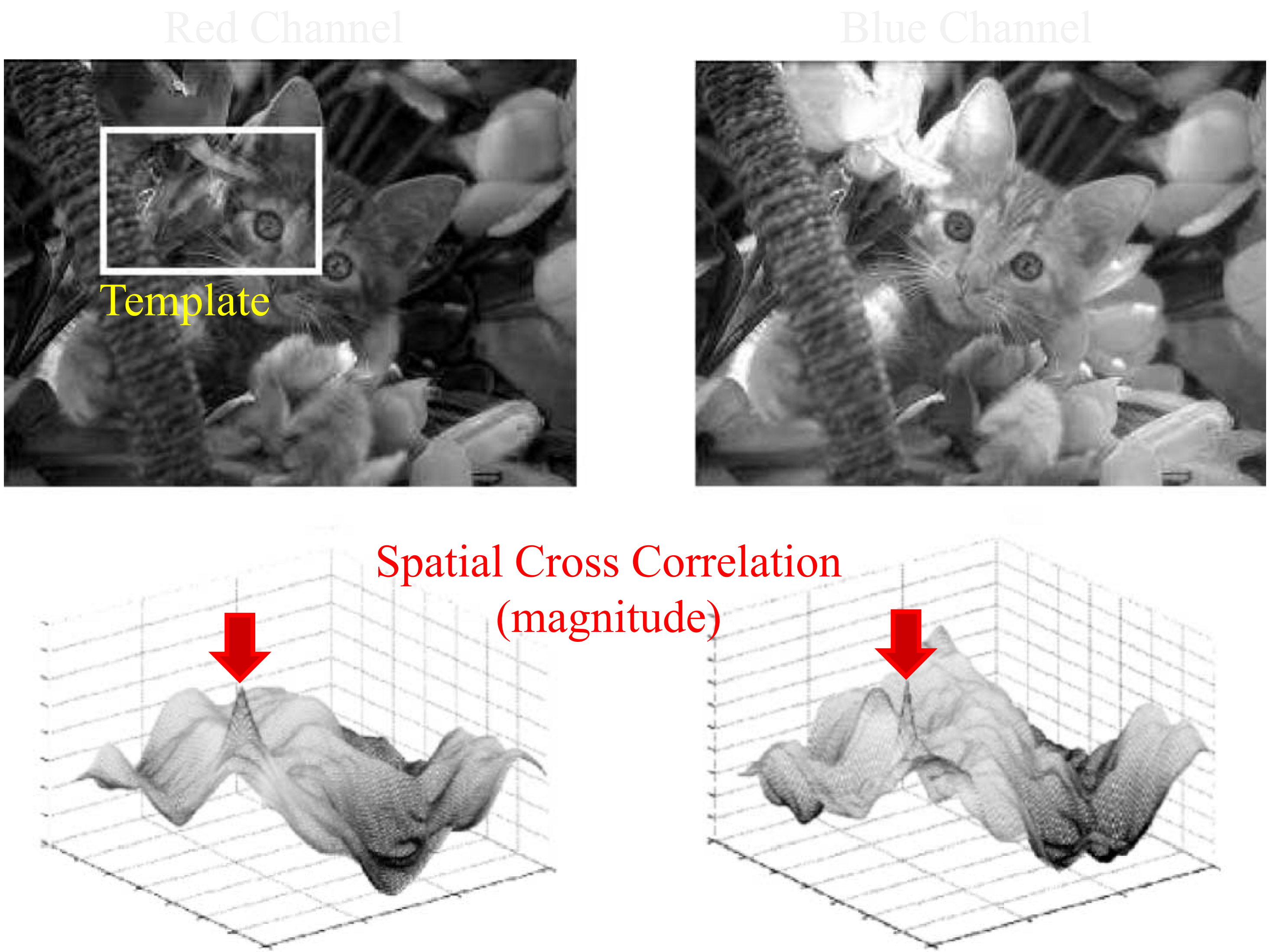
$$CC(x,y)=\iint f^{*}(p,q)\cdot g(p+x,q+y)\,dp\,dq$$

$$NCC(x,y)=\frac{1}{n-1}\sum_{x,y}\frac{(f(x,y)-\bar{f})(g(x,y)-\bar{g})}{\sigma_f\sigma_g}$$

# Normalized cross correlation



# Normalized cross correlation



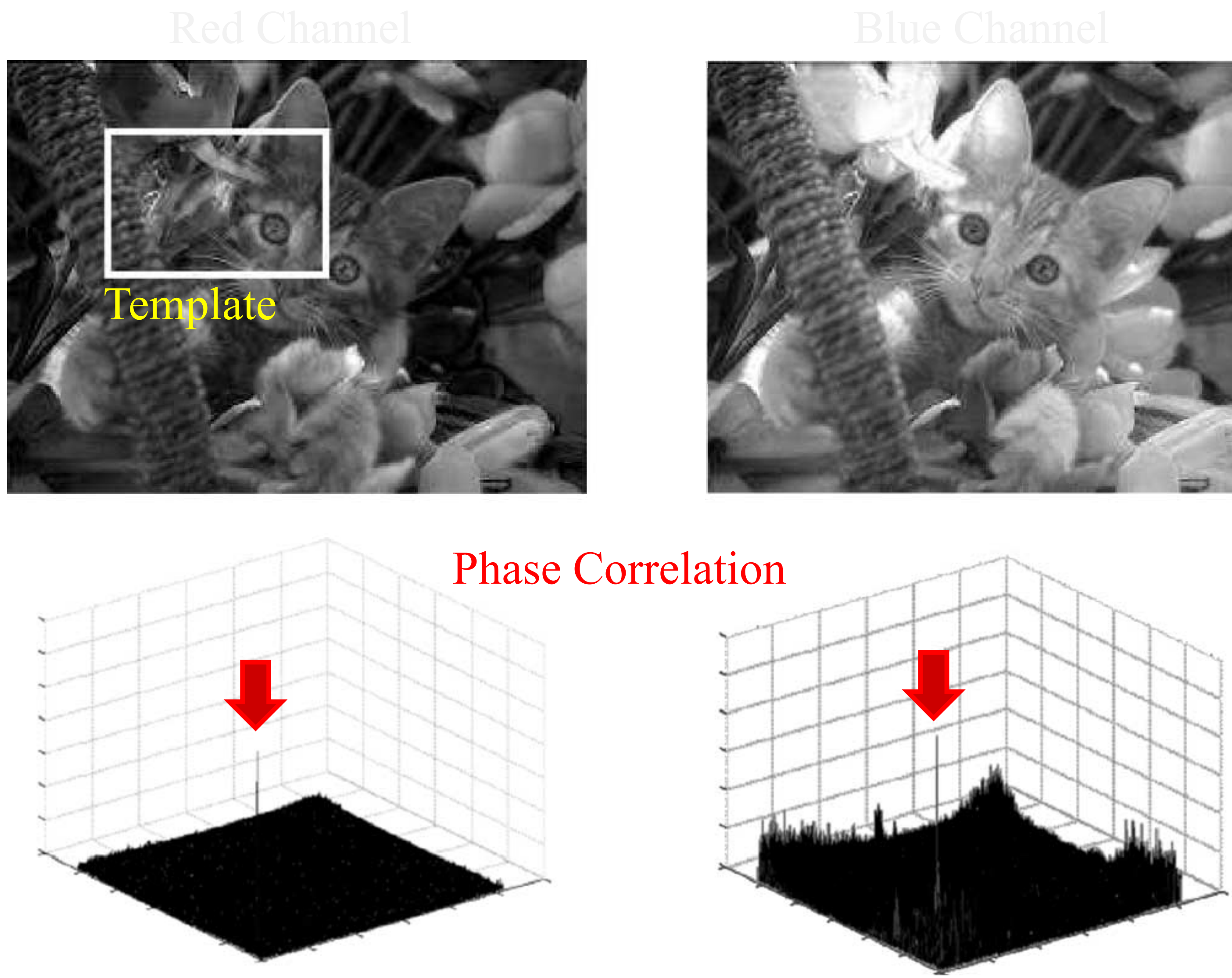
# Intensity-based methods (3)

- Fourier methods
  - Translation → phase correction in the frequency domain

$$\frac{F(f)F(g)^*}{\left|F(f)F(g)^*\right|} = e^{i2\pi(ux_0+vy_0)}$$

  - Time-saving for large images

# Phase correlation by Fourier methods



# Intensity-based methods (4)

- In information theory, entropy ($H$) is defined as a measure of uncertainty.
  - For an event $X$ with probability $P(X)$,

    $H(X) = -\Sigma P(x_i) [log_b(P(x_i))] = - \mathrm{Ex}[log_b(P(x_i))]$
  - When $P(X) = 0$ or 1, $H(X) = 0$ → No uncertainty at all
  - Example: toss a coin or fair dice
  - When $b = 2$, the unit of entropy is *bit*.

# Mutual information

- Mutual information
  - For two events (or images), *X* and *Y*,

$$\mathrm{MI}(X, Y) = \Sigma\, \Sigma\, P(x_i, y_j)\ log_b\, [(P(x_i, y_j)/P(x_i)P(\mathrm{y}_j)]$$
$$= H(X) + H(Y) - H(X, Y)$$

  - Representing the mutual dependency
  - Larger MI indicates better similarity.
  - Suitable for images obtained by different modalities

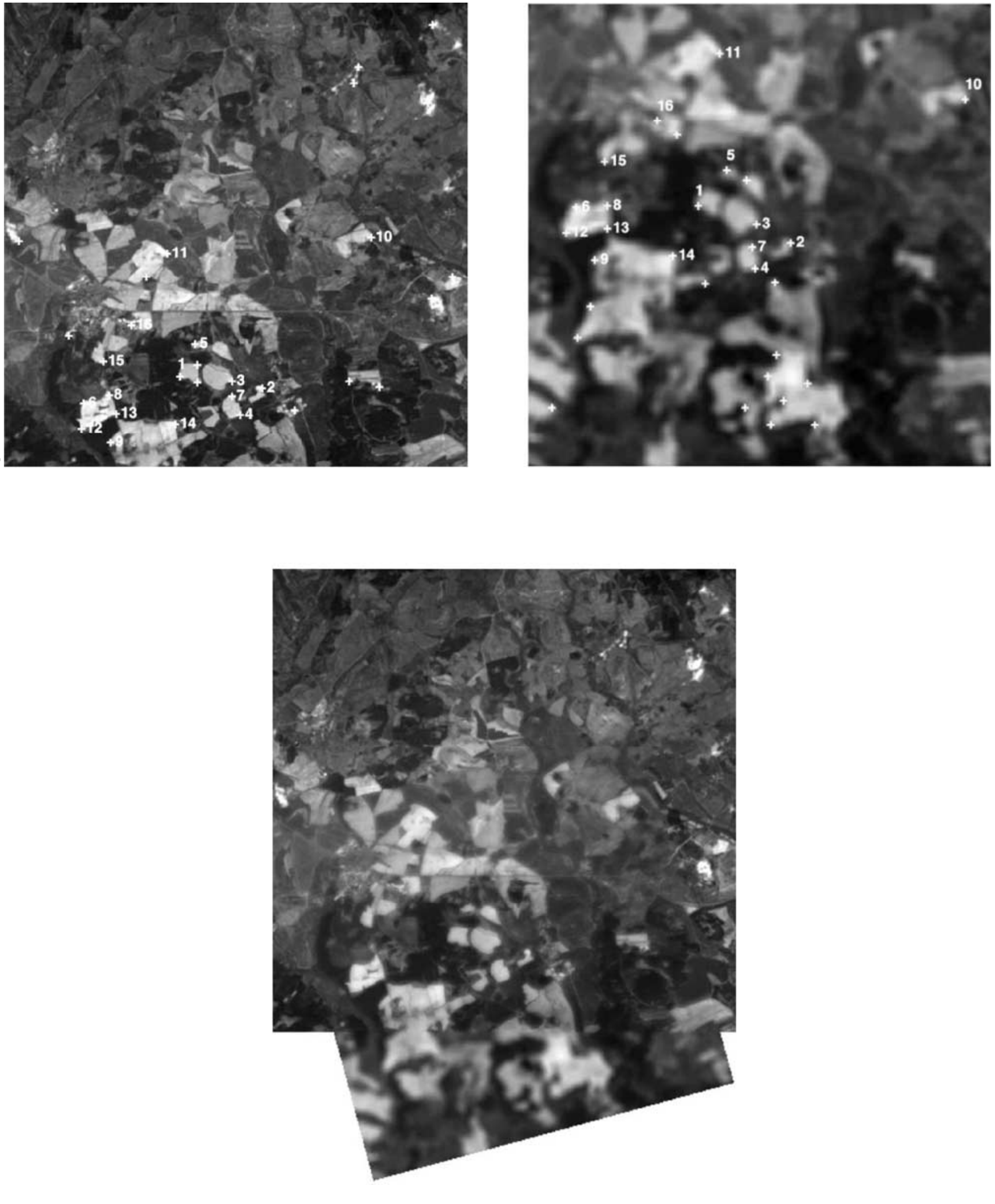
# Feature-based methods

- Features are defined/detected in both source and target images first.
  - Suitable for images containing enough distinctive and easily detectable features.
- Features can be
  - Points: center of gravity, corners, intersections of lines,…
  - Straight or curved lines: edges, textures,…
  - Regions: lakes, buildings,…

# Feature-based methods

- Methods using spatial relations
  - Distance between corresponding points/lines
- Methods using invariant descriptor
  - Invariant and unique features
  - Searching for best matching pairs in source and target images

# Feature-based method using invariant descriptors



# Resampling & Evaluation

# Image resampling

- According the constructed mapping function, the resource image is transformed to register.
- Forward manner:
  - Each pixel is directly transformed using the mapping function.
  - Cause holes and/or overlaps.
- Backward manner:
  - Looking for corresponding position on source image for each grid in the target.

# Backward approach of resampling

- Neither holes nor overlaps can occur.
- Require data interpolation
  - Convolution of the image with an interpolation kernel.
  - Interpolation algorithms include nearest neighbor, bilinear, bicubic,….

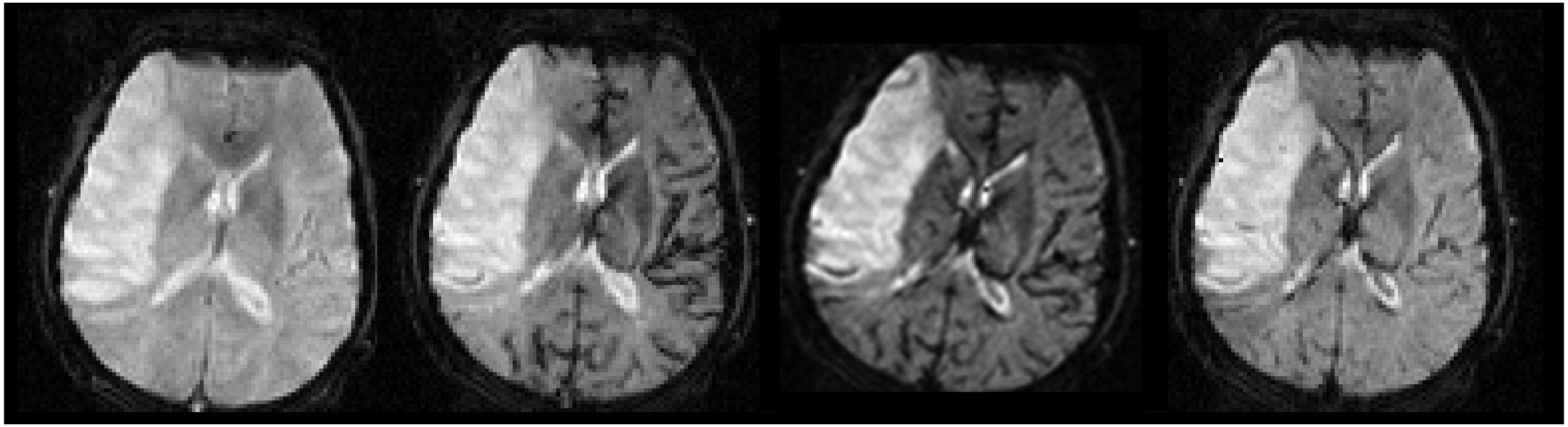
# Evaluation of registration accuracy

- Visual assessment by a domain expert
  - The oldest method, still in use
- Not a trivial problem because
  - Errors can be dragged into the registration process in any stage
  - Registration inaccuracies? Or actual physical differences?

# Applications of registration

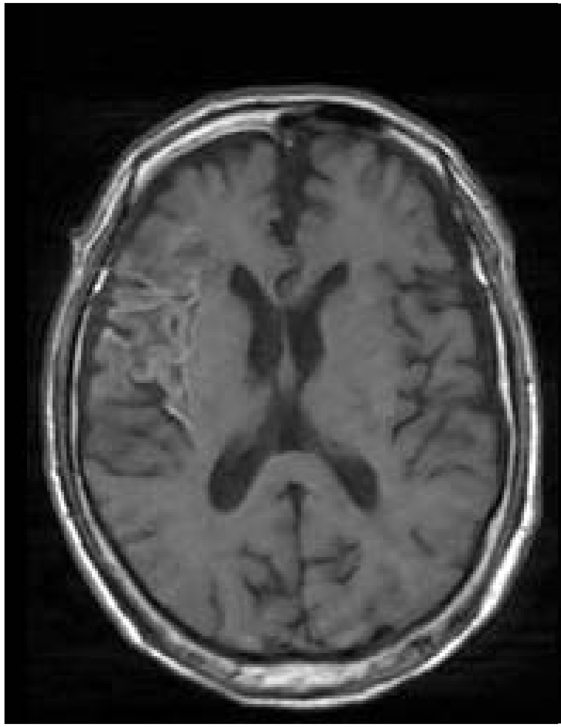
- Diagnosis based on all useful image information is necessary in clinical routine!
- Image registration plays an important role in scenarios including
  - Identical subject at different times
  - Identical subject at different modalities
  - Multiple subjects with same protocol

# EX: dynamic scans

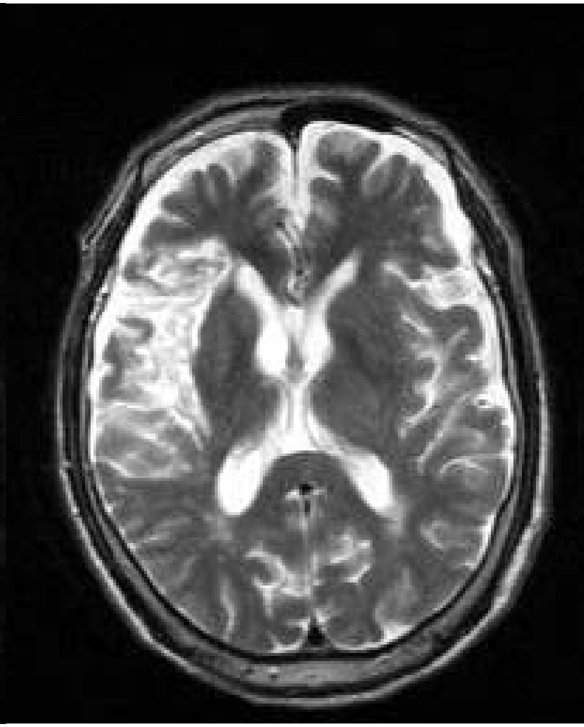


Dynamic contrast-enhanced MRI

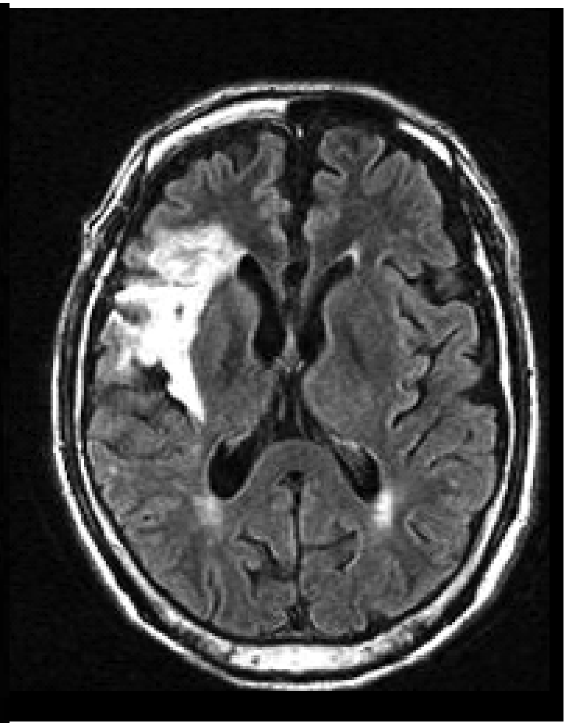
# EX: various contrasts



T1WI



T2WI

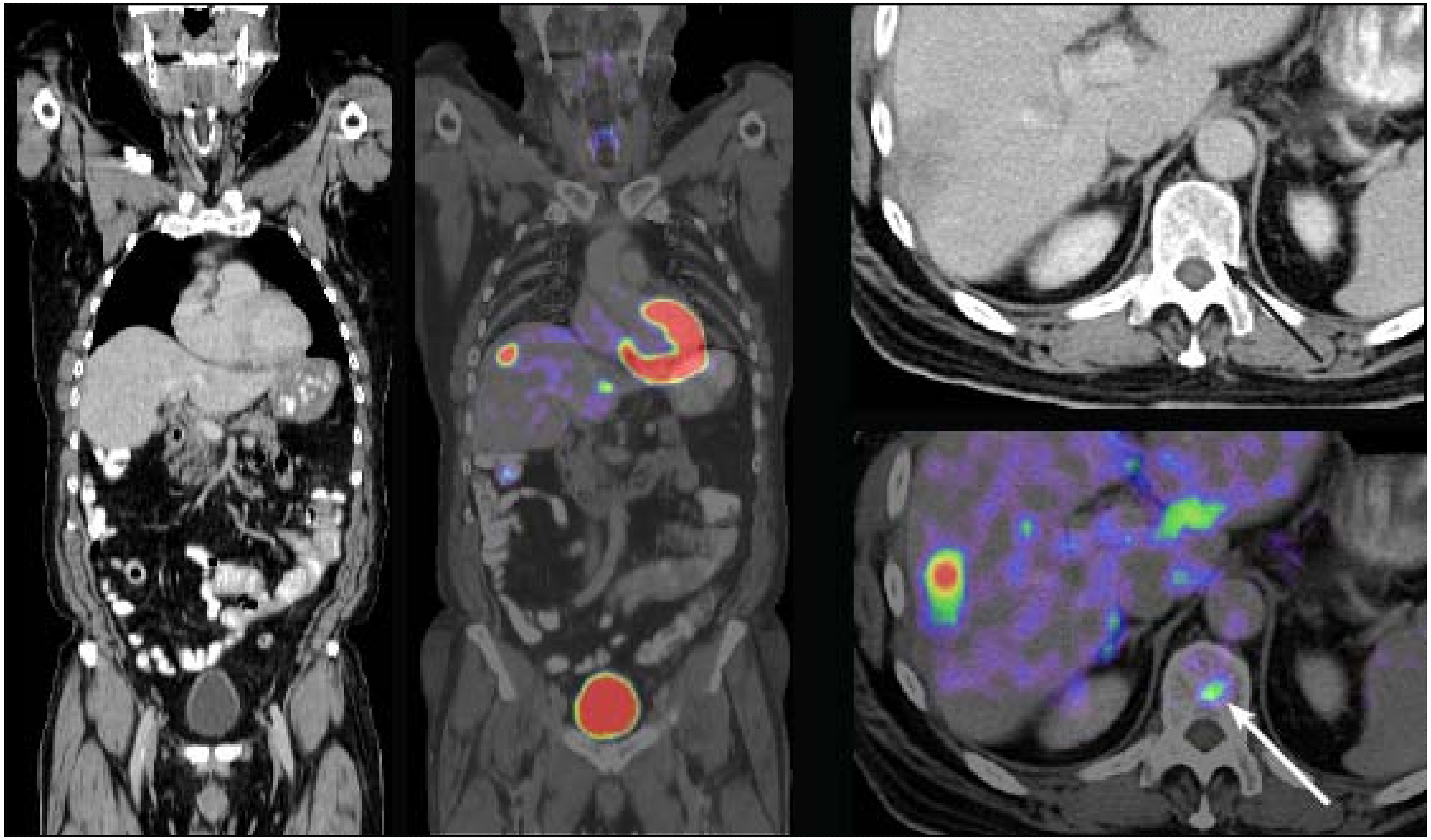


T2-FLAIR

# Applications of registration

- Identical subject at different times
- Identical subject at different modalities
  - CT + PET
  - MRI + PET
- Multiple subjects with same protocol

# EX: combine structural and metabolic info

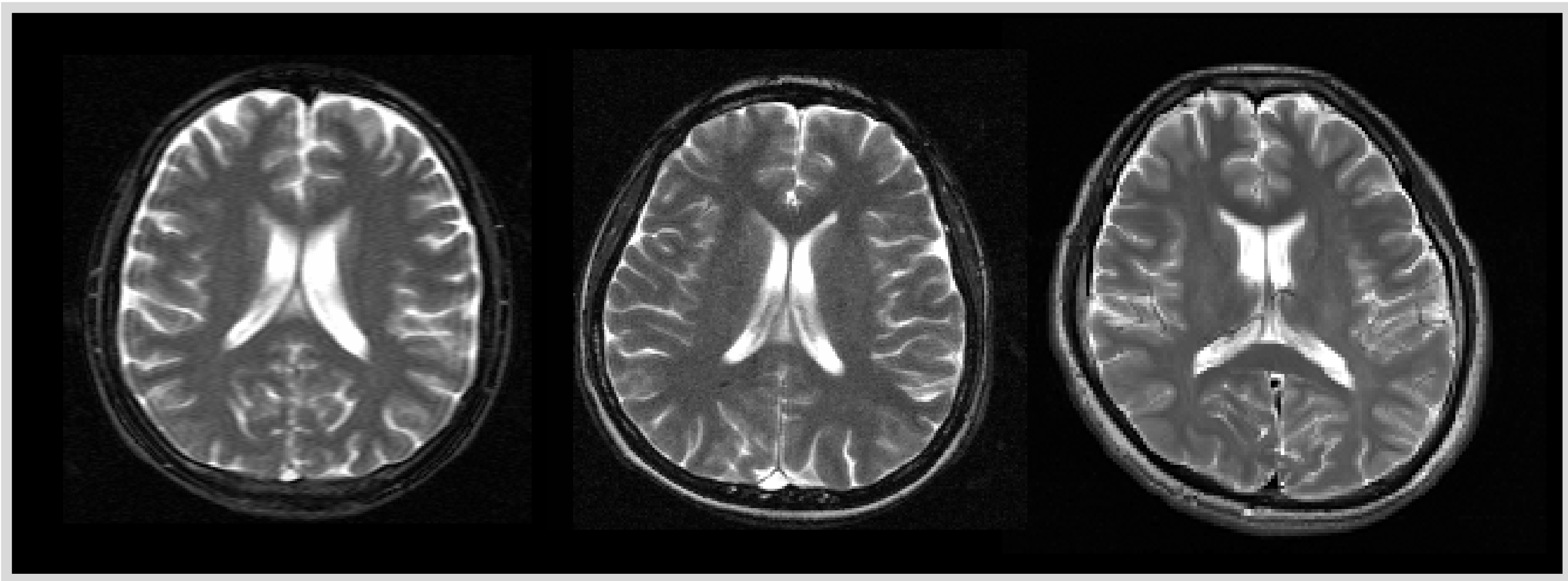


CT(gray-level) + PET (color) fusion

# Applications of registration

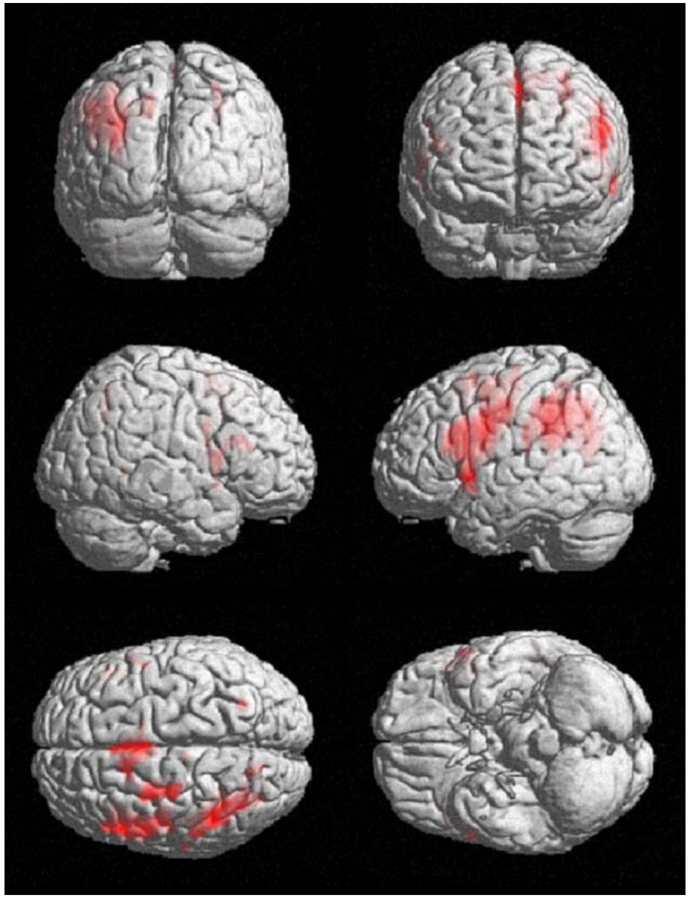
- Identical subject at different times
- Identical subject at different modalities
- Multiple subjects with same protocol
  - Group analysis

# EX: inter-subject comparison

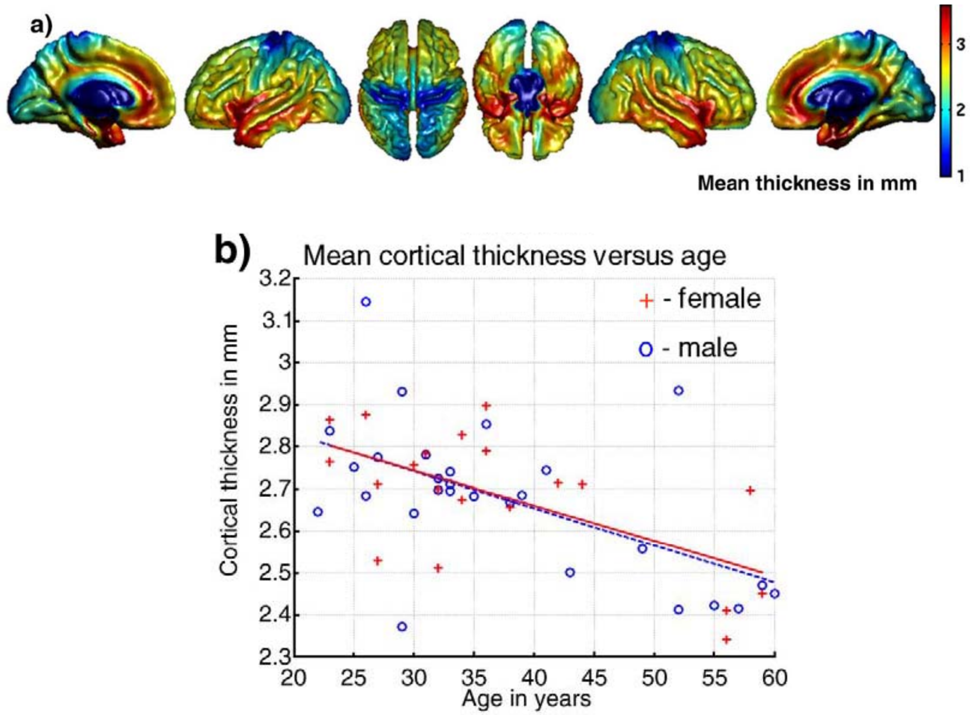


# EX: spatial normalization

fMRI



Aging in cortical thickness



Reference:

Dhawan, A.P., Medical image analysis (Chapter 9), John Wiley & Sons, 2002.

Zitova B. and Flusser J., “Image registration methods: a survey”, Image and vision computing 21(2003) p.977-1000.

Maintz J.B. and Viergever M.A., “A Survey of Medical Image Registration”, Med cal Image Analysis (1998) volume 2, number 1, pp 1–37

# 生醫影像研究方法：影像對位

# Example: Motion Correction

- Current Common Practice
  - e.g. SPM99
    - Transformation model : rigid (3 translations, 3 rotations)
    - Reference Image – a single T2* image
    - Similarity Metric: Sum of Squared Differences
- State of the Art
  - Integrated motion and distortion correction (recently in SPM8)
    - Transformation model : fully non-rigid
    - Reference Image – a single T2* image
    - Similarity Metric: Sum of Squared Differences